



IIT ROORKEE



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CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS

Lecture 34: Dispersion, Attenuation of Waves, Damping

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ERD of Foundations - Modules

1. Introduction
2. Shallow Foundations
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4. Well Foundations
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Module - 5

SSI for Deep Foundations



Module - 5: Chapters

1. Introduction to Soil-Structure Interaction
2. Effects of Soil-Structure Interaction
3. SGM and Wave Propagation
4. Dispersion & Attenuation of Waves, Damping
5. Ground Response Analysis
6. Soil-Pile Interaction

Chapter-4: Dispersion and Attenuation of Waves, Damping

- 1. Dispersion of Surface Waves**
- 2. Phase and Group Velocities**
- 3. Attenuation of Surface Waves**
- 4. Material and Radiation Damping of Soils**



Dispersion of Surface Waves

- The Rayleigh wave velocity in a homogeneous half-space is independent of frequency.
- The velocity of the Love wave, on the other hand, varies with frequency between an upper and a lower limit.
- ***Dispersion* is a phenomenon in which waves of different frequency (and different wavelength) propagate at different velocities.**
- The Love waves are *dispersive*, and Rayleigh waves in a homogeneous half-space are *nondispersive*.

Dispersion of Surface Waves

- Rayleigh waves of long wavelength (low frequency) can propagate faster than Rayleigh waves of short wavelength (high frequency). Therefore, in the real world of heterogeneous materials, Rayleigh waves are also dispersive.
- Since the velocities of both Rayleigh waves and Love waves decrease with increasing frequency, the low-frequency components of surface waves produced by earthquakes can be expected to arrive at a particular site before their high-frequency counterparts. This tendency to spread the seismic energy over time is an important effect of dispersion.

Phase and Group Velocities

- The solutions for Rayleigh wave velocity, V_R , and Love wave velocity, V_L , are based on the assumption of harmonic loading, which produces an infinite wave train.
- These velocities describe the rate at which points of constant phase (e.g., peaks, troughs, or zero points) travel through the medium and are called *phase velocities*.
- A transient disturbance may produce a packet of waves with similar frequencies. This packet of waves travels at the *group velocity*, C_g , given by

$$c_g = c + k \frac{dc}{dk}$$

Phase and Group Velocities

- Where c is the phase velocity (equal to V_R , or V_L , depending on which type of wave is being considered) and k is the wave number (equal to ω/V_R or ω/V_L).
- In a nondispersive material, $dc/dk = 0$, so the group velocity is equal to the phase velocity.
- Since both V_R and V_L generally decrease with increasing frequency in geologic materials, dc/dk is less than zero and the group velocity is lower than the phase velocity.

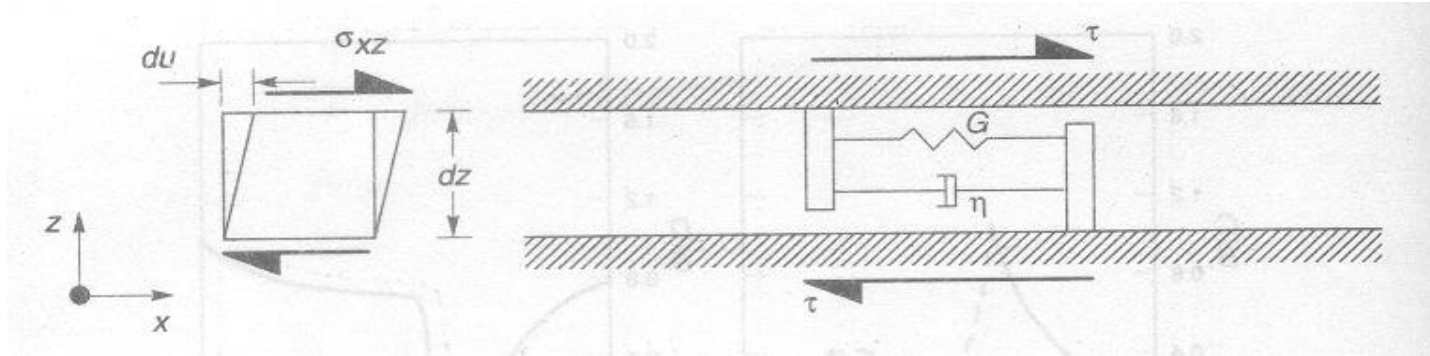
Attenuation of Surface Waves

- In a homogeneous linear elastic material, stress waves travel indefinitely without change in amplitude.
- This type of behavior cannot occur, however, in real materials. The amplitudes of stress waves in real materials, such as those that comprise the earth, attenuate with distance.
- This attenuation can be attributed to two sources, one of which involves the materials through which the waves travel and the other the geometry of the wave propagation problem.
 1. Material Damping
 2. Radiation Damping

Attenuation – Material Damping

- In real materials, part of the elastic energy of a traveling wave is always converted to heat. The conversion is accompanied by a decrease in the amplitude of the wave.
- Viscous damping, by virtue of its mathematical convenience, is often used to represent this dissipation of elastic energy.
- For the purpose of viscoelastic wave propagation, soils are usually modeled as Kelvin-Voigt solids (i.e., materials whose resistance to shearing deformation is the sum of an elastic part and a viscous part). A thin element of a Kelvin-Voigt solid can be illustrated as in Fig. (next slide).

Attenuation – Material Damping



Thin element of a Kelvin-Voigt solid subjected to horizontal shearing. Total resistance to shearing deformations is given by the sum of an elastic (spring) component and a viscous (dashpot) component.

Attenuation – Material Damping

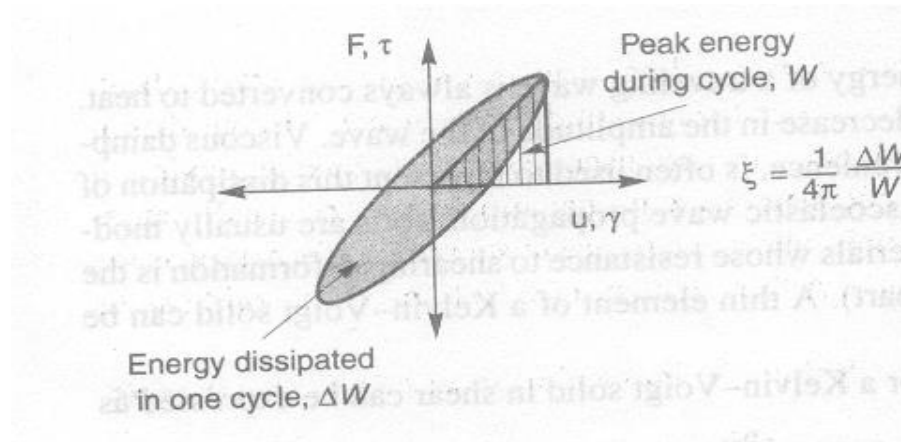
Thin element of a Kelvin-Voigt solid subjected to horizontal shearing. Total resistance to shearing deformations is given by the sum of an elastic (spring) component and a viscous (dashpot) component.

$$\tau = G\gamma + \eta \frac{\partial \gamma}{\partial t}$$

where $\tau (= \tau_{xz})$ is the shear stress, $\gamma (= \partial u / \partial z)$ is the shear strain, and η is the viscosity of the material. Thus the shear stress is the sum of an elastic part (proportional to strain) and a viscous part (proportional to strain rate).

Attenuation – Material Damping

For discrete Kelvin-Voigt systems, the damping ratio ξ , was shown to be related to the force-displacement (or, equivalently, the stress-strain) loop as shown in Fig.



Relationship between hysteresis loop and damping ratio

Attenuation – Material Damping

The peak energy stored in the cycle is.

$$W = \frac{1}{2} G \gamma_0^2$$

With these equations, damping ratio is given by

$$\xi = \frac{1}{4\pi} \frac{\Delta W}{W} = \frac{\eta \omega}{2G}$$

To eliminate frequency dependence while maintaining the convenience of the viscoelastic formulation, above Eq. is often rearranged to produce an equivalent viscosity that is inversely proportional to frequency. The use of this equivalent viscosity ensures that the damping ratio is independent of frequency.

Attenuation – Radiation Damping

- Since material damping absorbs some of the elastic energy of a stress wave, the *specific energy* (elastic energy per unit volume) decreases as the wave travels through a material.
- The specific energy can also decrease by another common mechanism, which is of pure geometric origin, resulting from the **decrease in specific energy that occurs as the area of the rod increases**.
- This reduction in amplitude due to spreading of the energy over a greater volume of material is often referred to as *radiation damping* (also as *geometric damping* and *geometric attenuation*).

Attenuation – Radiation Damping

- When earthquake energy is released from a fault below the ground surface, body waves travel away from the source in all directions. It can be shown that geometric attenuation causes **the amplitude to decrease at a rate of $1/r$** .
- The geometric attenuation of surface waves causes their amplitudes to **decrease at a rate of essentially $1/(\sqrt{r})$** ; in other words, surface waves attenuate (geometrically) much more slowly than body waves.
- This explains the greater proportion of surface wave motion (relative to body wave motion) that is commonly observed at large epicentral distances.

Attenuation – Radiation Damping

For problems in which energy is released from a finite source, ranging from the large-scale case of rupture along an earthquake fault to the smaller-scale case of a vibrating foundation, radiation damping can be extremely important. In such cases the effects of radiation damping often dominate those of material damping

References:

1. Prakash S. (1981), Soil Dynamics, McGraw-Hill Book Company, New York
2. Kramer S.L. (1996). Geotechnical Earthquake Engineering, Prentice Hall, Upper Saddle River, NJ, USA

Thank You

